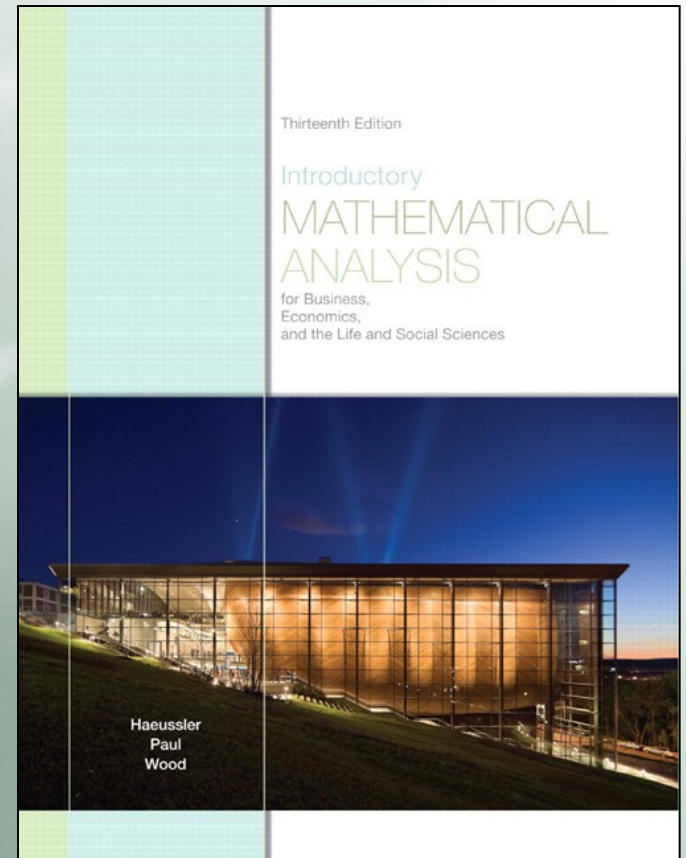


INTRODUCTORY MATHEMATICAL ANALYSIS

For Business, Economics, and the Life and Social Sciences

Chapter 1 Applications and More Algebra



Chapter Outline

- 1.1) Applications of Equations
- 1.2) Linear Inequalities
- 1.3) Applications of Inequalities
- 1.4) Absolute Value

0.7 Equations, in Particular Linear Equations

Equations

- An **equation** is a statement that two expressions are equal.
- The two expressions that make up an equation are called its **sides**.
- They are separated by the **equality sign**, $=$.

Example 1 - Examples of Equations

a. $x + 2 = 3$

b. $x^2 + 3x + 2 = 0$

c. $\frac{y}{y-4} = 6$

d. $w = 7 - z$

- A **variable** (e.g. x , y) is a symbol that can be replaced by any one of a set of different numbers.
- Solving equation means finding all possible values of the variables that make the equation true

Example 3 - Solving a Linear Equation

Solve $5x - 6 = 3x$.

Solution:

$$5x - 6 = 3x$$

$$5x - 6 + (-3x) = 3x + (-3x)$$

$$2x - 6 = 0$$

$$2x - 6 + 6 = 0 + 6$$

$$2x = 6$$

$$\frac{2x}{2} = \frac{6}{2}$$

$$x = 3$$

1.1 Applications of Equations

- **Modeling:** Translating relationships in the problems to mathematical symbols.

Example 1 - Mixture

A chemist must prepare 350 ml of a chemical solution made up of two parts alcohol and three parts acid. How much of each should be used?

Chapter 1: Applications and More Algebra

1.1 Applications of Equations

Example 1 - Mixture

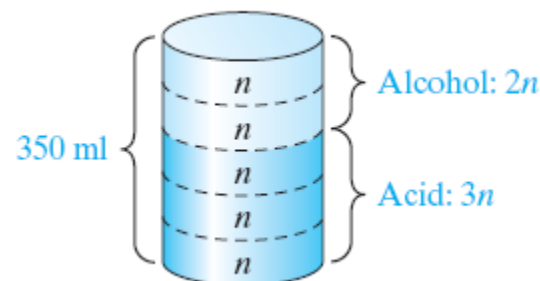
Solution:

Let n = number of milliliters in each part.

$$2n + 3n = 350$$

$$5n = 350$$

$$n = \frac{350}{5} = 70$$



Each part has 70 ml.

Amount of alcohol = $2n = 2(70) = 140$ ml

Amount of acid = $3n = 3(70) = 210$ ml

Applications of Equations in Economics

- **Fixed cost** is the sum of all costs that are independent of the level of production.
- **Variable cost** is the sum of all costs that are dependent on the level of output.
- **Total cost** = variable cost + fixed cost
- **Total revenue** = (price per unit) x
(number of units sold)
- **Profit** = total revenue – total cost

Example 3 - Profit

The Anderson Company produces a product for which the variable cost per unit is \$6 and the fixed cost is \$80,000. Each unit has a selling price of \$10. Determine the number of units that must be sold for the company to earn a profit of \$60,000.

Chapter 1: Applications and More Algebra

1.1 Applications of Equations

Example 3 - Profit

Solution:

Let q = number of sold units.

fixed cost = 80,000

variable cost = $6q$

total cost = $6q + 80,000$

total revenue = $10q$

Since profit = total revenue – total cost

$$60,000 = 10q - (6q + 80,000)$$

$$140,000 = 4q$$

$$35,000 = q$$

35,000 units must be sold to earn a profit of \$60,000.

Example 5 - Investment

A total of \$10,000 was invested in two business ventures, A and B. At the end of the first year, A and B yielded returns of 6% and 5.75 %, respectively, on the original investments. How was the original amount allocated if the total amount earned was \$588.75?

Chapter 1: Applications and More Algebra

1.1 Applications of Equations

Example 5 - Investment

Solution:

Let x = amount (\$) invested at 6%.

Let x = the amount allocated for Venture A

$10,000 - x$ = the amount allocated for

Venture B

$$(0.06)x + (0.0575)(10,000 - x) = 588.75$$

$$0.06x + 575 - 0.0575x = 588.75$$

$$0.0025x = 13.75$$

$$x = 5500$$

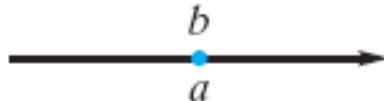
\$5500 was invested at 6% for Venture A

$\$10,000 - \$5500 = \$4500$ was invested at

5.75% for Venture B.

1.2 Linear Inequalities

- Supposing a and b are two points on the real-number line, the relative positions of two points are as follows:



$$a = b$$



$$a < b, a \text{ is less than } b$$
$$b > a, b \text{ is greater than } a$$

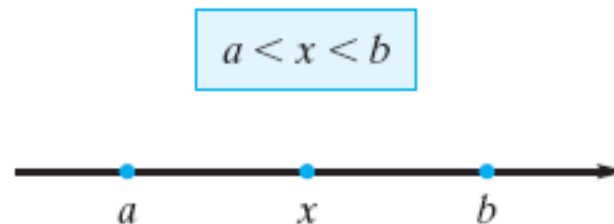


$$a > b, a \text{ is greater than } b$$
$$b < a, b \text{ is less than } a$$

- We use dots to indicate points on a number line.



- Suppose that $a < b$ and x is between a and b .



- **Inequality** is a statement that one number is less than another number.

Rules for Inequalities:

1.If $a < b$, then $a + c < b + c$ and $a - c < b - c$.

2.If $a < b$ and $c > 0$, then $ac < bc$
and $a/c < b/c$.

3.If $a < b$ and $c < 0$, then $a(c) > b(c)$ and $a/c > b/c$.

4.If $a < b$ and $a = c$, then $c < b$.

5.If $0 < a < b$ or $a < b < 0$, then $1/a > 1/b$.

6.If $0 < a < b$ and $n \geq 0$, then $a^n < b^n$.
 $\sqrt[n]{a} < \sqrt[n]{b}$

If $0 < a < b$, then .

- **Linear inequality** can be written in the form

$$ax + b < 0$$

where a and b are constants and $a \neq 0$

- To **solve** an inequality involving a variable is to find all values of the variable for which the inequality is true.

Example 1 - Solving a Linear Inequality

Solve $2(x - 3) < 4$.

Solution:

Replace inequality by equivalent inequalities.

$$2(x - 3) < 4$$

$$2x - 6 < 4$$

$$2x - 6 + 6 < 4 + 6$$

$$2x < 10$$

$$\frac{2x}{2} < \frac{10}{2}$$

$$x < 5$$

Example 3 - Solving a Linear Inequality

Solve $(3/2)(s - 2) + 1 > -2(s - 4)$.

Solution:

$$\frac{3}{2}(s - 2) + 1 > -2(s - 4)$$

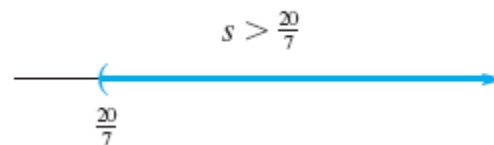
$$2\left[\frac{3}{2}(s - 2) + 1\right] > 2[-2(s - 4)]$$

$$[3(s - 2) + 2] > -4(s - 4)$$

$$3s - 4 > -4s + 16$$

$$7s > 20$$

$$s > \frac{20}{7}$$



The solution is
 $(\frac{20}{7}, \infty)$.

1.3 Applications of Inequalities

- Solving word problems may involve inequalities.

Example 1 - Profit

For a company that manufactures aquarium heaters, the combined cost for labor and material is \$21 per heater. Fixed costs (costs incurred in a given period, regardless of output) are \$70,000. If the selling price of a heater is \$35, how many must be sold for the company to earn a profit?

Example 1 - Profit

Solution:

$$\text{profit} = \text{total revenue} - \text{total cost}$$

Let q = number of heaters sold.

$$\text{total cost} = \text{fixed cost} + \text{variable cost}$$

$$\text{Total cost} = 70,000 + 21q$$

$$\text{total revenue} - \text{total cost} > 0$$

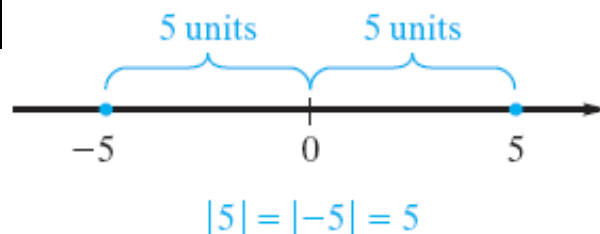
$$35q - (21q + 70,000) > 0$$

$$14q > 70,000$$

$$q > 5000$$

1.4 Absolute Value

- On real-number line, the distance of x from 0 is called the **absolute value** of x , denoted as $|x|$



DEFINITION

The **absolute value** of a real number x , written $|x|$, is defined by

$$|x| = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x < 0 \end{cases}$$

Example 1 - Solving Absolute-Value Equations

a. *Solve* $|x - 3| = 2$

b. *Solve* $|7 - 3x| = 5$

c. *Solve* $|x - 4| = -3$

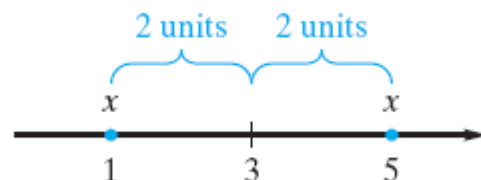
Chapter 1: Applications and More Algebra

1.4 Absolute Value

Example 1 - Solving Absolute-Value Equations

Solution:

$$\begin{array}{lcl} \text{a. } x - 3 = 2 & \text{or} & x - 3 = - \\ & & x = 1 \end{array}$$



$$\begin{array}{lcl} \text{b. } 7 - 3x = 5 & \text{or} & 7 - 3x = -5 \\ & & x = 4 \end{array}$$

c. The absolute value of a number is never negative. The solution set is $\emptyset$.

Absolute-Value Inequalities

- Summary of the solutions to absolute-value inequalities is given.

Inequality ($d > 0$)	Solution
$ x < d$	$-d < x < d$
$ x \leq d$	$-d \leq x \leq d$
$ x > d$	$x < -d$ or $x > d$
$ x \geq d$	$x \leq -d$ or $x \geq d$

Example 3 - Solving Absolute-Value Equations

a. Solve $|x + 5| \geq 7$

b. Solve $|3x - 4| > 1$

Solution:

$$\begin{array}{lcl} \text{a. } x + 5 \leq -7 & \text{or} & x + 5 \geq -7 \\ & & x \leq -12 \qquad \qquad x \geq -2 \end{array}$$

We write it as $(-\infty, -12] \cup [-2, \infty)$, where $\cup$ is the *union* symbol.

$$\begin{array}{lcl} \text{b. } 3x - 4 < -1 & \text{or} & 3x - 4 > 1 \\ & & x < 1 \qquad \qquad x > \frac{5}{3} \end{array}$$

We can write it as $(-\infty, 1) \cup \left(\frac{5}{3}, \infty\right)$.

Properties of the Absolute Value

- 5 basic properties of the absolute value:

1. $|ab| = |a| \cdot |b|$

2. $\left| \frac{a}{b} \right| = \frac{|a|}{|b|}$

3. $|a - b| = |b - a|$

4. $-|a| \leq a \leq |a|$

5. $|a + b| \leq |a| + |b|$

- Property 5 is known as *the triangle inequality*.

Example 5 - Properties of Absolute Value

Solution

:

a. $|(-7) \cdot 3| = |-7| \cdot |3| = 21$

b. $|4 - 2| = |2 - 4| = 2$

c. $|7 - x| = |x - 7|$

d. $\left| \frac{-7}{3} \right| = \frac{|-7|}{|3|} = \frac{7}{3}; \left| \frac{-7}{-3} \right| = \frac{|-7|}{|-3|} = \frac{7}{3}$

e. $\left| \frac{x-3}{-5} \right| = \frac{|x-3|}{|-5|} = \frac{|x-3|}{5}$

f. $-|2| \leq 2 \leq |2|$

g. $|(-2) + 3| = |1| = 1 \leq 5 = 2 + 3 = |-2| + |3|$